energy equation. It is shown in present numerical analysis that the phase function and optical radii have significant effect on heat flux distribution of the problem considered.

Acknowledgments

This research was supported by the National Science Council of the Republic of China through Grants NSC 79-0401-E009-14.

References

[1]Tong, T. W., McElroy, D. L., and Yarbrough, D. W., "Transient Conduction and Radiation Heat Transfer in Porous Thermal Insulation," *Journal of Thermal Insulation,* Vol. 9, July 1985, pp. 13–29.

[2]Barker, C., and Sutton, W. H., "The Transient Radiation and Conduction Heat Transfer in a Gray Participating Medium with Semi-Transparent Boundaries," *ASME Radiation Heat Transfer,* edited by B. F. Armaly and A. F. Emery, HTD-Vol. 49, 1985, pp. 25–36.

[3]Sparrow, E. M., Usiskin, C. M., and Huibbard, H. A., "Radiation Heat Transfer in a Spherical Enclosure Contained in Participating, Heat-Generation Gas," *Journal of Heat Transfer,* Vol. 83, No. 2, 1961, pp. 199–206.

[4]Ryhming, I. L., "Radiative Transfer Between Two Concentric Spheres Separated by an Absorbing and Emitting Gas," *International Journal of Heat and Mass Transfer,* Vol. 9, April 1966, pp. 315–324.

[5]Viskanta, R., and Merriam, R. L., "Heat Transfer by Combined Conduction and Radiation Between Concentric Spheres Separated by Radiation Medium," *Journal of Heat Transfer,* Vol. 90, May 1968, pp. 248–256.

[6]Thynell, S. T., and Özisik, M. N., "Radiation Transfer in an Isotropically Scattering Homogeneous Solid Sphere," *Journal of Quantitative Spectroscopy and Radiative Transfer,* Vol. 33, No. 4, 1985, pp. 383–395.

[7]Thynell, S. T., "Interaction of Conduction and Radiation in Anisotropically Scattering Spherical Media," *Journal of Thermophysics and Heat Transfer,* Vol. 4, No. 3, 1990, pp. 299–304.

[8]Tong, T. W., and Swathi, P. S., "Radiative Heat Transfer in Emitting-Absorbing-Scattering Spherical Medium," *Journal of Thermophysics and Heat Transfer,* Vol. 1, No. 2, 1987, pp. 162–170.

[9]Li, W., and Tong, T. W., "Radiative Heat Transfer in Isothermal Spherical Media," *Journal of Quantitative Spectroscopy and Radiative Transfer,* Vol. 43, No. 3, 1990, pp. 239–251.

[10]Bayazitoglu, Y., and Higenyi, J., "Higher-Order Differential Equations of Radiative Transfer: P_3 Approximation," *AIAA Journal,* Vol. 17, No. 4, 1979, pp. 424–431.

Backward Monte Carlo Modeling for Rocket Plume Base Heating

H. F. Nelson*

University of Missouri—Rolla, Rolla, Missouri 65401

Introduction

RADIATIVE base heating of a rocket due to its exhaust plume has been investigated actively for several years. Reviews of the analysis of base heating prior to 1966 are available.[1,2] Methods for making practical calculations have been developed.[3–6]

Radiation emitted from a participating medium with a conical geometry is difficult to analyze compared to slab, or cylindrical geometries. Gorshkova[7] used a zonal method to solve for the radiation in a truncated cone with isotropic scattering and constant properties. Kaminski[8] used both a Monte Carlo and a P-1 approximation to investigate radiative heat transfer from a nonscattering medium with uniform properties in a truncated cone enclosure. Lin and Wang[9] calculated the radiation from an absorbing, isotropic scattering, conical medium. They investigated the influence of cone angle and cone length on the radiation heating of the exterior base region. Their geometry was similar to that for the plume radiation heating of the base of a rocket.

Studies of plume radiation have been performed for conical geometries. Bobco[10,11] and Edwards[12] developed a plume model to predict base heating. The radiation was described in terms of radiosity that varied axially along the plume. Edwards and Bobco[13] and Edwards et al.[14] also formulated an engineering model to predict the radiosity of conical plumes. Watson and Lee[15] used a Monte Carlo model to calculate the radiation from solid rocket plumes. The model accounted for axial and radial property variations of both the particles and the gases in the plume. The scattering was allowed to be either isotropic or anisotropic.

The purpose of this note is 1) to show that the backward Monte Carlo method can be used to predict radiative heating for geometries similar to those for rocket plume base heating, and 2) to compare the backward Monte Carlo predictions with previous work done by other methods. The motivation for this research is to develop a computer code to calculate the base heating of a rocket due to its plume radiation. The backward Monte Carlo model seems to be the best way to calculate the radiative base heating from realistic plumes.

Formulation

Figure 1 shows a schematic of the rocket plume and base heating geometry. The origin of the coordinate system is at the center of the rocket nozzle exit plane. The base of the rocket is located at $z = -z_0$, where z_0 is the length of the rocket nozzle. The radiation flux from a point $\boldsymbol{x}_p$ on the plume to a point $\boldsymbol{x}_0$ on the base is defined as

$$F_\lambda(\boldsymbol{x}_0) = \int_{\Delta\Omega} \varepsilon_\lambda(\boldsymbol{x}_p, \theta', \phi') I_{b\lambda}(T_{\text{ref}}) \cos\theta \, d\Omega \qquad (1)$$

where, $\varepsilon_\lambda(\boldsymbol{x}_p, \theta', \phi')$ is the radiation emission coefficient, θ is the angle between the vector $\boldsymbol{n}_r$, along the line connecting points $\boldsymbol{x}_0$ and $\boldsymbol{x}_p$ and the outward normal vector $\boldsymbol{n}_0$ at $\boldsymbol{x}_0$ on the rocket base, $\Delta\Omega$ is the solid angle of the plume as viewed from the point $\boldsymbol{x}_0$ and θ' and ϕ' represent the polar and azimuthal angular orientation of the radiation intensity ray with respect to an outward normal vector $\boldsymbol{n}_p$ at point $\boldsymbol{x}_p$. The reference Planck intensity is $I_{b\lambda}(T_{\text{ref}})$, where T_{ref} is the reference temperature (normally the maximum temperature in the plume). Note that the medium between points $\boldsymbol{x}_0$ and $\boldsymbol{x}_p$ is noninteracting. In the Monte Carlo analysis it is assumed that the photon does not lose any energy as it travels from its point of emission in the plume, to the plume boundary; hence, the emission at $\boldsymbol{x}_p$ is directly related to the temperature

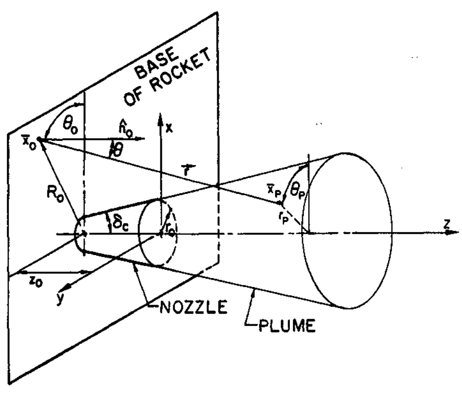

Fig. 1 Schematic of the rocket plume geometry.

Received June 5, 1991; revision received Aug. 26, 1991; accepted for publication Aug. 26, 1991.

* Professor of Aerospace Engineering, Thermal Radiative Transfer Group, Department of Mechanical and Aerospace Engineering and Engineering Mechanics. Associate Fellow AIAA.

at x. The incremental solid angle can be written as $\Delta\Omega = \Delta A_{\perp}/(r^2)$, where, $\Delta A_{\perp}$ is the component of the differential plume surface area at x_p perpendicular to n_r.

Backward Monte Carlo Method

Equation (1) is solved using the backward Monte Carlo method. This method uses less computer time than direct Monte Carlo. It is applicable when one is interested in radiation at a specific point, from a specific direction. In the backward Monte Carlo method photons initially travel in a direction opposite to that which they physically propagate in the plume. A photon is incident at a point on the plume along the ray in the direction in which one wants to calculate the radiation emitted from the plume. The photon path is followed through the plume from event to event as the photon interacts with the gases and particles. Eventually, the photon is either absorbed in the plume, or escapes through the plume boundary. Photons that escape are assumed to be lost, unless they escape into the rocket exhaust nozzle; consequently, contributing to searchlight emission. Photons that are absorbed are assumed to be emitted at the point where they were absorbed with power equal to $I_{b\lambda}(T(x))$, where $T(x)$ is the temperature at the point x where the absorption event occurred. The emitted photon then retraces its incident path forward in time and it eventually emerges from the plume at the point at which it was originally incident on the plume, but in the opposite direction. In this case the integrand of Eq. (1) can be written as

$$I_{b\lambda}(T_{\text{ref}})\varepsilon_\lambda(x_p, \theta', \phi') = \frac{\sum_{n=1}^{N_{\text{abs}}} [I_{b\lambda}(T(x))]_n}{N_{\max}}$$

$$= \frac{[I_{b\lambda}(T(x_1)) + \cdots + I_{b\lambda}(T(x_{N_{\text{abs}}}))]}{N_{\max}} \tag{2}$$

where N_{abs} is the number of photons absorbed in the plume and $N_{\max}$ is the total number of incident photons. The sum in the numerator contains one term for each photon emitted in the plume. Each absorbed photon is weighted by the Planck intensity function evaluated at $T(x)$. Note that Eq. (2) reduces to $\varepsilon_\lambda(x_p, \theta', \phi') = N_{\text{abs}}/N_{\max}$ for emission from an isothermal plume. For the limting case of gray radiation, one has $\varepsilon_\lambda(x_p, \theta', \phi') = \varepsilon(x_p, \theta', \phi')$. Using this in Eq. (1) and integrating over all wavelengths one obtains

$$\frac{F(x_0)}{\sigma T_{\text{ref}}^4} = \frac{1}{\pi}\int_{\Delta\Omega} \varepsilon(x_p, \theta', \phi')\cos\theta \, d\Omega \tag{3}$$

For an isothermal, gray plume $\varepsilon(x_p, \theta', \phi') = N_{\text{abs}}/N_{\max}$.

Searchlight Emission

Searchlight emission is an important aspect of plume radiation and it may contribute significantly to plume signature and to plume base heating. It is composed of the radiation emitted in the rocket nozzle and combustion chamber. The photons propagate axially out the nozzle and are scattered out of the plume in a direction in such a way that they contribute to the signature, or base heating. The searchlight emission is easily accounted for in the backward Monte Carlo method. All the photons that enter the nozzle are absorbed, either by the gas-particle mixture, or by the nozzle walls, and reemitted in such a way that they all contribute to searchlight emission.

Results and Discussion

Comparison to Previous Results

The backward Monte Carlo computer code was verified by comparing its predictions to previously published results in the literature for geometries similar to the rocket base heating geometry. The first comparisons were with Tien and Abu-Romia,[16] who predicted the radiation from simple plume-like geometries. The plumes were assumed to be isothermal, to have constant radiative properties, to be nonscattering ($\sigma = 0$) and to have no emission from the nozzle exit plane. The backward Monte Carlo predictions are shown to agree well with those of Tien and Abu-Romia in Ref. 17.

Figure 2 shows a comparison of the backward Monte Carlo calculations with the pure scattering ($\kappa = 0$), cylindrical plume (plume length $= 10\ r_0$) results of Stockham and Love.[18] It shows the nondimensional radiative flux $F(x_0)/(\sigma T_{\text{ref}}^4)$, as a function of R_0/r_0 on the baseplane. In this case, the nozzle exit plane has a finite temperature T_{ref}, while the scattering medium has zero absolute temperature. The scattering is assumed to be isotropic. The radiation emitted from the nozzle exit plane is scattered by the particles in the plume and eventually some of it strikes the baseplate at R_0/r_0. This situation models the searchlight problem. The agreement of the backward Monte Carlo predictions with the Stockham and Love[18] calculations is very good for $\tau_{r_0} = \beta_{r_0} = 0.50$. ($\beta = \sigma + \kappa$, is the extinction coefficient and $\omega = \sigma/\beta$ is the albedo.)

These comparison cases are physically very different. One is a pure absorbing, emitting plume, while the other is a pure scattering, nonemitting plume. In both cases, the agreement between the backward Monte Carlo and the published results is good. The greatest difference occurs at large R_0/r_0, where the plume emission reaching R_0/r_0 is very low and the Monte Carlo statistics deteriorate.

Effect of Plume Albedo and Cone Angle

Figures 3 and 4 show the radiation reaching point R_0/r_0 on the rocket base as a function of cone angle δ_c for plumes with $\tau_{r_0} = 0.50$ and albedos of 0.90 and 0.99, respectively. The calculations were done assuming an isothermal, constant property plume with a length of 50 cm and $r_0 = 1$ cm. The total radiation at R_0/r_0 is the sum of the plume emission and the searchlight emission. The searchlight radiation is assumed to be emitted with the same power as the plume radiation.

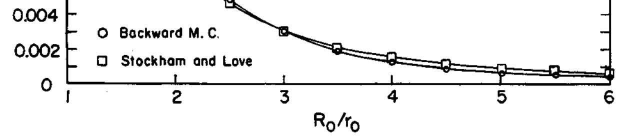

Fig. 2 Heating of rocket base due to a cylindrical, isothermal, nonabsorbing, isotropic scattering, plume. Comparison of Stockham and Love results, □, and backward Monte Carlo results ○ for $\omega = 1.0$, $\tau_{r_0} = 0.5$ and $z_0 = 0$.

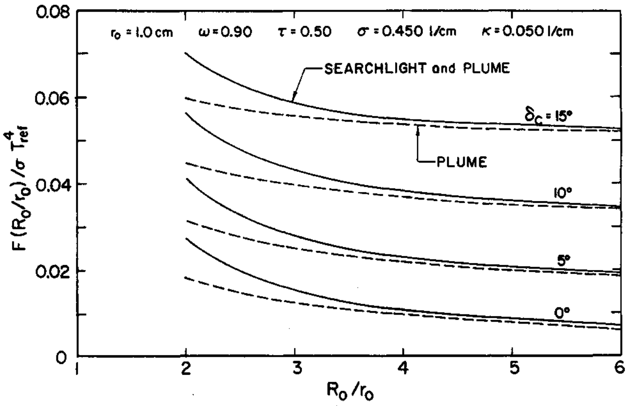

Fig. 3 Heating of rocket base due to a conical, isothermal, isotropic scattering, $\omega = 0.90$ plume. Effect of searchlight emission and plume cone angle for $\tau_{r_0} = 0.5$ and $z_0 = 0$.

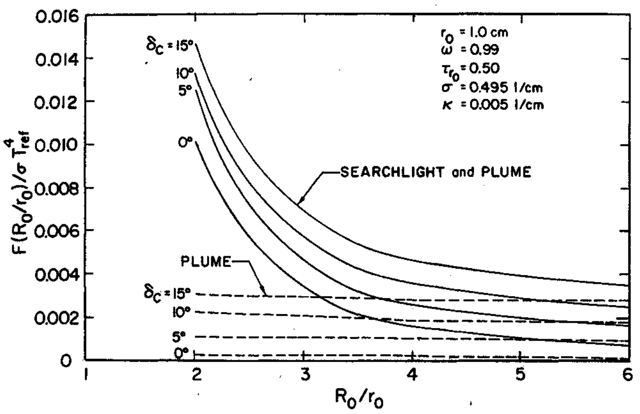

Fig. 4 Heating of rocket base due to a conical, isothermal, isotropic scattering, $\omega = 0.99$ plume. Effect of searchlight emission and plume cone angle for $\tau_{r_0} = 0.5$ and $z_0 = 0$.

In other words, the temperature of the plume in the nozzle is the same as the temperature of the exhaust plume. If the temperature in the nozzle were twice the plume temperature, the searchlight emission would increase by a factor of 16. The figures show searchlight emission can be very important. The percentage of heating due to searchlight emission decreases as both δ_c and R_0/r_0 increase.

As the plume albedo increases toward one, the plume emission decreases and the searchlight emission increases. In the limit of $\omega = 1$, the radiation at R_0/r_0 will be entirely due to searchlight effects (see Fig. 2). The general trends are that as δ_c increases, radiative heating increases and as ω increases radiative heating decreases. Searchlight radiation can become important for highly scattering, low emitting plumes at small R_0/r_0.

The computer run time varies with the number of photons that are input per grid point on the plume surface and with the number of grid points. A typical run time to calculate the radiation at one R_0/r_0 position is 20 s on an IBM 4381 computer, for 200 photons/(grid point) and 200 grid points (40,000 photons) for the results presented herein.

Conclusions

Backward Monte Carlo calculations work well for radiation base heating predictions and the predictions agree well with previously published results for similar problems. For isothermal, gray plumes the radiative base heating increases as plume cone angles increase. As the plume albedo increases from zero toward one, the radiative heating decreases. Searchlight radiative heating becomes important as the plume albedo increases. It is generally more important at small values of R_0/r_0 and decreases as R_0/r_0 increases. In more realistic, variable property plumes, the albedo is a local value, but the same trends will occur. As the scattering becomes more significant the average albedo increases and searchlight radiation will become more important.

Acknowledgments

This work was partially supported by Remtech Inc., Huntsville, Alabama. The backward Monte Carlo code was obtained from Robert A. Reed, Sverdrup Technology, Inc., Arnold Air Force Base, TN.

References

[1]Rochelle, W. C., "Review of Thermal Radiation from Liquid and Solid Propellant Rocket Exhausts," NASA TMX 53579, Feb. 1967.

[2]Carlson, D. J., "Radiation from Rocket Exhaust Plumes, Part II: Metalized Solid Propellants," AIAA 2nd Propulsion Joint Specialist Conference, Colorado Springs, CO, AIAA Paper 66-652, June 13–17, 1966.

[3]Fontenot, J. E., Jr., "Thermal Radiation from Solid Rocket Plumes at High Altitude," *AIAA Journal*, Vol. 3, No. 5, 1965, pp. 970–972.

[4]Morizumi, S. J., and Carpenter, H. J., "Thermal Radiation from the Exhaust Plume of an Aluminized Composite Propellant Rocket," *Journal of Spacecraft and Rockets*, Vol. 1, No. 5, 1964, pp. 501–507.

[5]Bartky, C. D., and Bauer, E., "Predicting the Emittance of a Homogeneous Plume Containing Alumina Particles," *Journal of Spacecraft and Rockets*, Vol. 3, No. 10, 1966, pp. 1523–1527.

[6]Bobco, R. P., and Edwards, R. H., "Radiation from an Absorbing Scattering Conical Dispersion with Non-Uniform Density," Aerospace Technology Research Report, SSD 60571R, Hughes Aircraft Co., Los Angeles, CA, 1966.

[7]Gorshkova, S. N., "Radiative Heat Transfer in a Truncated Cone Chamber in which the Heat Source Lies within the Gas Space," *Heat Transfer—Soviet Research*, Vol. 18, No. 1, 1986, pp. 39–43.

[8]Kaminski, D. A., "Radiative Heat Transfer from a Gray, Absorbing-Emitting Medium in a Conical Enclosure," *Proceedings of the 1988 American Society of Mechanical Engineers National Heat Transfer Conference*, HTD-Vol. 96, 1988, pp. 201–207.

[9]Lin, J. D., and Wang, C. S., "Radiative Heat Transfer from an Absorbing and Isotropically Scattering Medium Contained within a Truncated Cone," *American Society of Mechanical Engineers National Heat Transfer Conference*, HTD-Vol. 106, Heat Transfer Phenomena in Radiation, Combustion and Fires, edited by R. K. Shah, 1989, pp. 205–211.

[10]Bobco, R. P., "Radiation from Conical Surfaces with Nonuniform Radiosity," *AIAA Journal*, Vol. 4, No. 3, 1966, pp. 544–546.

[11]Bobco, R. P., "Reply by Author to D. K. Edwards on Radiation from Conical Surfaces with Nonuniform Radiosity," *AIAA Journal*, Vol. 7, No. 8, 1969, p. 1659.

[12]Edwards, D. K. "Comment on Radiation from Conical Surfaces with Nonuniform Radiosity," *AIAA Journal*, Vol. 7, No. 8, 1969, pp. 1656–1658.

[13]Edwards, D. K., and Bobco, R. P., "Effect of Particle Size Distribution on the Radiosity of Solid Propellant Rocket Motor Plumes," Spacecraft Radiative Transfer and Temperature Control, edited by T. E. Horton, Vol. 83, Progress in Astronautics and Aeronautics, AIAA, New York, 1982, pp. 169–188.

[14]Edwards, D. K., Sakurai, Y., and Babikan, D. S., "A Two-Particle Model for Rocket Plume Radiation," *Journal of Thermophysics and Heat Transfer*, Vol. 1, No. 1, 1987, pp. 13–20.

[15]Watson, G. H., and Lee, A. L., "Thermal Radiation Model for Solid Rocket Booster Plumes," *Journal of Spacecraft and Rockets*, Vol. 14, No. 11, 1977, pp. 641–647.

[16]Tien, C. L., and Abu-Romia, M. M., "A Method of Calculating Rocket Plume Radiation to the Base Region," *Journal of Spacecraft and Rockets*, Vol. 1, No. 4, July–Aug., 1964, pp. 433–435.

[17]Nelson, H. F., "Modeling for Rocket Plume Base Heating Calculations," 19th JANNAF Plume Technology Conference, U.S. Army Missile Command, Redstone Arsenal, AL, May 14–16, 1991. Chemical Propulsion Information Agency, CPIA Publication 568, May 1991, pp. 585–592.

[18]Stockham, L. W., and Love, T. J., "Radiative Heat Transfer from a Cylindrical Cloud of Particles," *AIAA Journal*, Vol. 6, No. 10, 1968, pp. 1935–1940.

Free Convection About Vertical Needles Embedded in a Saturated Porous Medium

Chunmei Peng,* Taofang Zeng†, and Gang Chen‡
Huazhong University of Science and Technology, Wuhan, People's Republic of China

Nomenclature

a = size of needle
C = constant

Received April 1, 1991; revision received Sept. 4, 1991; accepted for publication Sept. 5, 1991.

*Assistant Professor, Department of Power Engineering; currently, Engineer of Wuhan Boiler Works.

†Graduate Student; Department of Engineering Mechanics, Tsinghua University, Beijing.

‡Teaching Assistant; currently, Ph.D. Candidate, Department of Mechanical Engineering, University of California, Berkeley.